RELATIONSHIP BETWEEN THE COMPACT COMPLEX AND REAL VARIABLE 2-D FDTD METHODS IN ARBITRARY ANISOTROPIC DIELECTRIC WAVEGUIDES

An Ping Zhao, Jaakko Juntunen, and Antti V. Räisänen

Radio Laboratory
Helsinki University of Technology, FIN-02150, Espoo, Finland

Abstract

The relationship between the compact complex and real variable 2-D FDTD methods used for the analysis of guided modes of arbitrary anisotropic dielectric waveguides is investigated. Situations for the permittivity tensor with different non-zero elements are discussed. It is found that in certain cases the complex 2-D FDTD method cannot be reduced to the real variable one. This, in turn, reveals that the real variable 2-D FDTD method has limitation when applied to arbitrary anisotropic dielectric waveguides. In addition, numerical results show that using the complex impulse in the excitation is not an essential condition, even for a *purely* complex 2-D FDTD method.

Introduction: Since the two-dimensional Finite Difference Time Domain (FDTD) method was developed [1, 2], it has been widely used for the analysis of the guided modes of different waveguides. Reviewing the development history of the 2-D FDTD technique, an original 2-D FDTD approach based on a non-truly 2-D mesh (i.e., a two-dimensional mesh in the cross-section combined with one-half mesh in the propagating direction of the guide) was first developed [1]. In parallel, a compact complex 2-D FDTD method, which exactly uses a truly 2-D mesh and therefore significantly improves the efficiency of the non-truly 2-D FDTD approach, was proposed [2] and its stability was further investigated in [3]. Due to the attractive feature of the compact 2-D FDTD technique [2, 3], nowadays it is being used more than the original method [1]. Nevertheless, the compact 2-D FDTD approaches proposed in [2, 3] are based on the processing of the complex variable and such a processing was claimed [4] as one of the disadvantages of the method. In order to overcome this disadvantage (i.e., avoiding the complex variable) and to further improve the efficiency of the compact complex 2-D FDTD method, a compact real variable 2-D FDTD algorithm was developed [4]. The advantage of the real variable 2-D FDTD method [4] over the complex 2-D FDTD method [2] is obvious since only half of the computer memory and CPU time are required in the real variable algorithm. Although it was claimed in [4] that the real variable 2-D FDTD technique can be applied to arbitrary anisotropic cases, we find that this is not always true. Theoretical analysis and numerical results presented in this paper show that, in certain circumstances (e.g., when the tangential field components (E_x and/or E_y) are coupled with the longitudinal field component (E_z)), the complex 2-D FDTD method cannot be reduced to (or replaced by) the real variable 2-D FDTD method, thus the complex 2-D FDTD method has to be employed in these cases; and in this paper such a complex 2-D FDTD method is regarded as the *purely* complex 2-D FDTD method.

Theory: Assuming that for the guided modes supported by an anisotropic dielectric waveguide with an arbitrary relative permittivity tensor, [ε], the field variation along the axis of the waveguide, z, is of the form exp(-jβz), where β is the propagation constant and $j = \sqrt{-1}$. Thus, Maxwell's equations can be written as:

$$\frac{\partial H_x^{re}}{\partial t} = \frac{1}{\mu_0}\left(-\frac{\partial E_z^{re}}{\partial y} + \beta E_y^{im}\right) \qquad (1.1a)$$

$$\frac{\partial H_y^{re}}{\partial t} = \frac{1}{\mu_0}\left(\frac{\partial E_z^{re}}{\partial x} - \beta E_x^{im}\right) \qquad (1.2a)$$

$$\frac{\partial H_z^{im}}{\partial t} = \frac{1}{\mu_0}\left(\frac{\partial E_x^{im}}{\partial y} - \frac{\partial E_y^{im}}{\partial x}\right) \quad (1.3a)$$

$$\frac{\partial D_x^{im}}{\partial t} = \frac{\partial H_z^{im}}{\partial y} + \beta H_y^{re} \quad (1.4a)$$

$$\frac{\partial D_y^{im}}{\partial t} = -\frac{\partial H_z^{im}}{\partial x} - \beta H_x^{re} \quad (1.5a)$$

$$\frac{\partial D_z^{re}}{\partial t} = \frac{\partial H_y^{re}}{\partial x} - \frac{\partial H_x^{re}}{\partial y} \quad (1.6a)$$

$$\frac{\partial H_x^{im}}{\partial t} = \frac{1}{\mu_0}\left(-\frac{\partial E_z^{im}}{\partial y} - \beta E_y^{re}\right) \quad (1.1b)$$

$$\frac{\partial H_y^{im}}{\partial t} = \frac{1}{\mu_0}\left(\frac{\partial E_z^{im}}{\partial x} + \beta E_x^{re}\right) \quad (1.2b)$$

$$\frac{\partial H_z^{re}}{\partial t} = \frac{1}{\mu_0}\left(\frac{\partial E_x^{re}}{\partial y} - \frac{\partial E_y^{re}}{\partial x}\right) \quad (1.3b)$$

$$\frac{\partial D_x^{re}}{\partial t} = \frac{\partial H_z^{re}}{\partial y} - \beta H_y^{im} \quad (1.4b)$$

$$\frac{\partial D_y^{re}}{\partial t} = -\frac{\partial H_z^{re}}{\partial x} + \beta H_x^{im} \quad (1.5b)$$

$$\frac{\partial D_z^{im}}{\partial t} = \frac{\partial H_y^{im}}{\partial x} - \frac{\partial H_x^{im}}{\partial y} \quad (1.6b)$$

where μ_0 is the permeability in vacuum; the superscripts 're' and 'im' represent the real part and the imaginary part of the field components, respectively; and both the real part and the imaginary part are real variables. To easily handle arbitrary anisotropic dielectric media, the magnetic field, **H**, the electric field, **E**, and the electric displacement, **D**, are together involved in Eq. (1). Because only anisotropic dielectric materials are considered here, all elements of $[\varepsilon]$ are real and $[\varepsilon]$ is a symmetric matrix. Therefore, the relationship between the (complex) electric field (**E**) and the (complex) electric displacement (**D**) is of the form:

$$\begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} = \frac{1}{\varepsilon_0}[\varepsilon]^{-1}\begin{bmatrix} D_x \\ D_y \\ D_z \end{bmatrix} = \frac{1}{\varepsilon_0}\begin{bmatrix} \varepsilon_{xx}\varepsilon_{xy}\varepsilon_{xz} \\ \varepsilon_{yx}\varepsilon_{yy}\varepsilon_{yz} \\ \varepsilon_{zx}\varepsilon_{zy}\varepsilon_{zz} \end{bmatrix}^{-1}\begin{bmatrix} D_x \\ D_y \\ D_z \end{bmatrix}$$

$$= \frac{1}{\varepsilon_0}\begin{bmatrix} \varepsilon_{xx}\varepsilon_{xy}\varepsilon_{xz} \\ \varepsilon_{xy}\varepsilon_{yy}\varepsilon_{yz} \\ \varepsilon_{xz}\varepsilon_{yz}\varepsilon_{zz} \end{bmatrix}^{-1}\begin{bmatrix} D_x \\ D_y \\ D_z \end{bmatrix} \quad (2)$$

where ε_0 is the permittivity in vacuum, and $[\varepsilon]^{-1}$ is the inverse matrix of $[\varepsilon]$. In what follows, the relationship between the compact complex and real variable 2-D FDTD methods will be discussed for three possible anisotropic dielectric cases.

Case 1: The permittivity tensor has only the diagonal elements, i.e., $\varepsilon_{ij} \equiv 0$ for $i \neq j$. By combining Eqs. (1) with (2), one can see that the 12 equations in Eq. (1) can be separated into two independent groups A and B (the real variable field components (H_x^{re}, H_y^{re}, H_z^{im}, E_x^{im}, E_y^{im}, E_z^{re}) are involved in group A, whereas in group B the real variable field components (H_x^{im}, H_y^{im}, H_z^{re}, E_x^{re}, E_y^{re}, E_z^{im}) are concerned) since these two groups are not coupled. Because of such an independent behavior, for the guided modes supported by this special anisotropic waveguide it is unnecessary to solve *all* 12 equations of Equation (1). Alternatively, the propagation characteristics of the guided modes can be obtained by using either group A or group B, i.e., for this case the complex 2-D FDTD method [2] can be reduced to the real variable 2-D FDTD method [4].

Case 2: If an off diagonal case, i.e., $\varepsilon_{xy} \neq 0$ and $\varepsilon_{xz} = \varepsilon_{yz} \equiv 0$, is considered, then the 12 equations in Eq. (1) can still be separated into the two independent groups A and B. Thus, for the same reason as explained in Case 1, the complex 2-D FDTD method

[2] can again be reduced to the real variable 2-D FDTD method [4] for Case 2.

Case 3: However, if another off diagonal situations [5], e.g., either both ε_{xz} and ε_{yz} are not zero or one of them is non-zero (noting that in this case the choice of ε_{xy} is arbitrary - it can be either zero or non-zero), are considered, then the real variable 2-D FDTD method is *no longer* valid. This is due to the fact that in this case the tangential field components (E_x and/or E_y) are coupled with the longitudinal field component (E_z). Thus, once Eqs. (1) and (2) are combined, the 12 equations in Eq. (1) *cannot* be separated into two independent groups. This certainly indicates that for the situations considered in Case 3 the complex 2-D FDTD method [2] *cannot* be replaced by the real variable 2-D FDTD method [4], i.e., the complex 2-D FDTD method has to be employed.

Numerical Validation: Because the validation for Case 1 and Case 2 has been done in [4], therefore only Case 3 will be considered here. In order to prove the above theory, an anisotropic square waveguide (W = t) whose optics axis (c) lies in the yz-plane at an angle $\theta = 45°$ from the z axis (in this case $\varepsilon_{yz} \neq 0$ and thus it belongs to Case 3) is considered [5]. Even through for this example the purely complex 2-D FDTD method has to be employed, we use only a real impulse (e.g., $D_x^{re} \times \exp(-(n\Delta t - t_0)^2/T^2)$ is used for the E_{11}^x mode; and $D_y^{re} \times \exp(-(n\Delta t - t_0)^2/T^2)$ is used for the E_{11}^y mode), instead of the complex impulse [2, 3], in the excitation. Figure 1 shows the dispersion characteristics of the E_{11}^x and E_{11}^y modes of the anisotropic square waveguide obtained from the complex 2-D FDTD method. A comparison with [5] shows an excellent agreement. To further demonstrate the theory developed above, the contour plot of the actual E_y ($= |E_y^{re} + j E_y^{im}|$) field component is illustrated in Fig. 2; whereas the surface plots of the real (E_y^{re}) and imaginary (E_y^{im}) parts of the E_y field component of the E_{11}^x mode are shown in Figs. 3(a) and 3(b), respectively. It can be seen from Figs. 3(a) and 3(b) that the imaginary part of the field component can still be obtained even only the real impulse is used in the excitation. Furthermore, it is not surprised that the profile of the actual E_y field component shown in Fig. 2 is not symmetric along the y direction because for the case under consideration the optics axis lies in the yz-plane and $\varepsilon_{yz} \neq 0$ [5]. Finally, if one examines the distributions of the E_y^{re} and E_y^{im} field components shown in Figs. 3 (a) and 3(b), due to the fact that the E_y field component is strongly coupled with the E_z field component, significant difference between them is observed.

Conclusions: By extending the 2-D FDTD method to arbitrary anisotropic dielectric materials, the relationship between the complex 2-D FDTD method and the real variable 2-D FDTD method is investigated for anisotropic waveguides with arbitrary tensor permittivity. It was found that, for the cases that the tangential field components (E_x and/or E_y) are coupled with the longitudinal field com-ponent (E_z), the complex 2-D FDTD method has to be employed, i.e., in these cases the complex 2-D FDTD method cannot be reduced to the real variable 2-D FDTD method. On the other hand, numerical results indicate that the real impulse can still be used in the excitation even for the purely complex 2-D FDTD method, and thus, the efficiency of the purely complex algorithm is further enhanced. By clearing up the relationship between the compact complex and real variable 2-D FDTD methods in our mind, as a consequence, the compact 2-D FDTD technique is now ready to study guide modes of arbitrary anisotropic dielectric waveguides.

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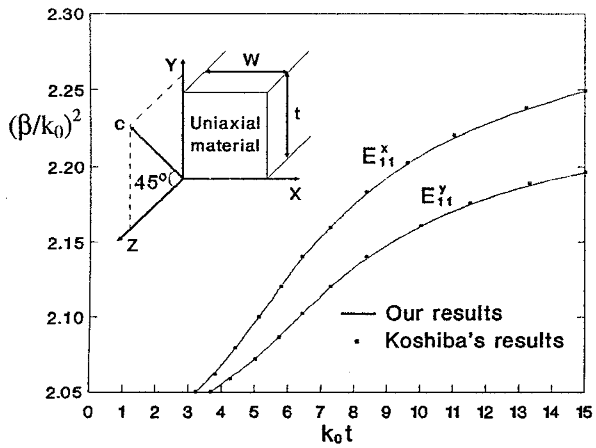

Figure 1. Dispersion characteristics of the E^x_{11} and E^y_{11} modes of the square anisotropic waveguide. $\Delta x = \Delta y = W/12$, and the total mesh dimension is 52×52.

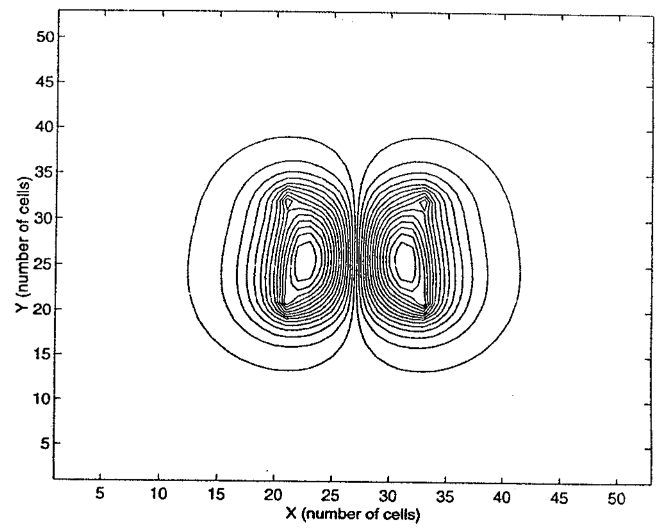

Figure 2. Contour plot of the actual E_y field component of the E^x_{11} mode of the waveguide ($W = t = 2.0$ μm) at $\beta = 7.416\times10^6\,m^{-1}$.

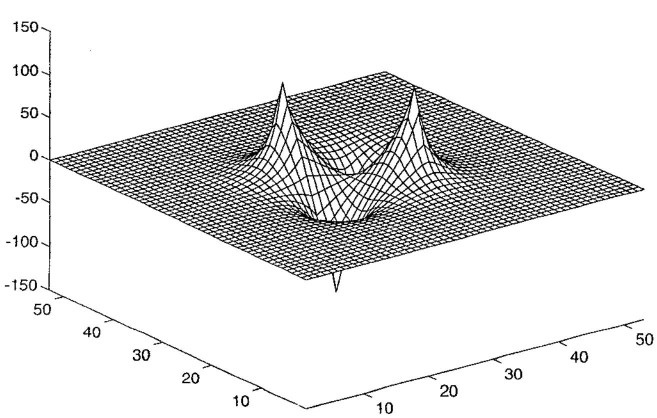

Figure 3(a). Surface plot of the E_y^{re} field component of the E^x_{11} mode of the waveguide.

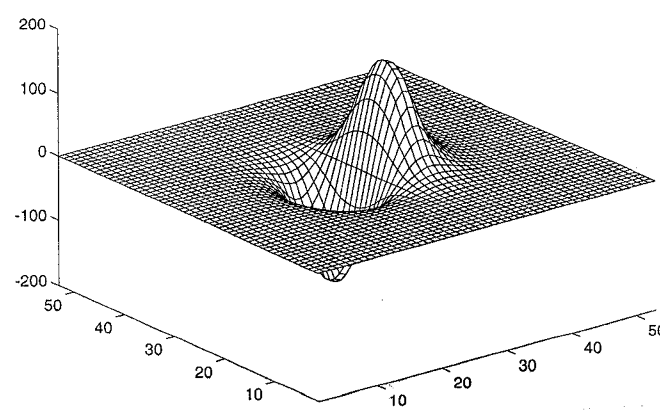

Figure 3(b). Surface plot of the E_y^{im} field component of the E^x_{11} mode of the waveguide.

An Ping Zhao (M'96) was born in Siping, China, on March 16, 1963. He received his BSc degree in optical physics in 1984 from the Institute of Optics and Fine Mechanics, Changchun, China, MSc degree in optics in 1987 from the Changchun Institute of Physics, Chinese Academy of Sciences, and PhD in electronic and electrical engineering in 1994 from Brunel Uni-versity, England.

From 1987 to 1989, he worked as an Associate Researcher at the Changchun Institute of Physics. In January 1990, he joined the Department of Electronic and Electrical Engineering, Surrey University, England, as a Visiting Research Fellow, where he worked on the computer verification of optical waveguide characteristics based on Multiple-Quantum-Well (MQW) structures in III-V semi-conductor materials. In October 1991, he became a PhD student at Surrey University. In October 1992, he moved to the Department of Electrical Engineering and Electronics, Brunel University, for continuing his PhD study. Since September 1994, he has been working as a Research Engineer at the Radio Laboratory, Helsinki University of Technology, Finland. His current research interests are the numerical modeling of integrated microwave circuits, using the FDTD method.

Jaakko Juntunen was born in Tornio, Finland, on March 27, 1971. He received his Master of Science degree in technical mathematics from the Helsinki University of Technology (HUT), Finland, in 1995.

From 1993 to 1995, he worked as a research assistant and as an acting assistant in mathematics and applied mathematics at the Institute of Mathematics, HUT. He made his Diploma Thesis about the development of high-order FEM for shell problems at the Institute of Mathematics, HUT. From the beginning of 1996 he moved to the Radio Laboratory, HUT, and since then has been working as a research engineer there. His main research interest is the developing of numerical models in computational electromagnetics, in the point of view of microwave and mm-wave applications.

Antti V. Räisänen (S'76-M'81-SM'85-F'94) received the Diploma Engineer (MSc), the Licentiate of Technology, and the Doctor of Technology degrees in electrical engineering from the Helsinki University of Technology (HUT), Finland, in 1973, 1976, and 1981, respectively.

From 1973 to 1987, he worked as a research assistant at the HUT Radio Laboratory. From 1978 to 1979, he was a research assistant at the Five College Radio Astronomy Observatory (FCRAO) of the University of Massachusetts, Amherst. From 1980 to 1983, he was a research fellow of the Academy of Finland, working mainly at HUT, but also for shorter periods at FCRAO and the Chalmers University of Technology, Gothenbury, Sweden. In 1984, he was a visiting scientist at the Department of Physics of the University of California, Berkeley. From 1985 to 1989, he was an acting professor of radio engineering with HUT, working also for short periods at the University of California, Berkeley. In 1989, he was appointed by invitation to the Professor Chair of Radio Engineering with HUT. In 1992-1993 he was on sabbatical leave with HUT and had a senior research fellowship from the National Research Council at the Jet Propulsion Laboratory, Pasadena, CA. He was also a visiting associate in electrical engineering at the California Institute of Technology. He is supervising research in millimeter components, antennas and receivers, microwave propagation in satellite links, microwave measurements, ect. at HUT Radio Laboratory. He has authored and co-authored more than 200 scientific or technique papers and three books: *Microwave Measurement Techniques*, *Radio Engineering, and RF and Microwave Techniques* (in Finnish).

Dr. Räisänen was the Secretary of the 12th European Microwave Conference, 1982. He was the Counselor of the IEEE Student Branch in Helsinki from 1982 to 1989, and the Chairman of the IEEE MTT/AP Chapter in Finland from 1987 to 1992. In 1992, he served as the Conference Chairman for the 22nd European Microwave Conference.

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